Accelerating Online CP Decompositions for Higher Order Tensors

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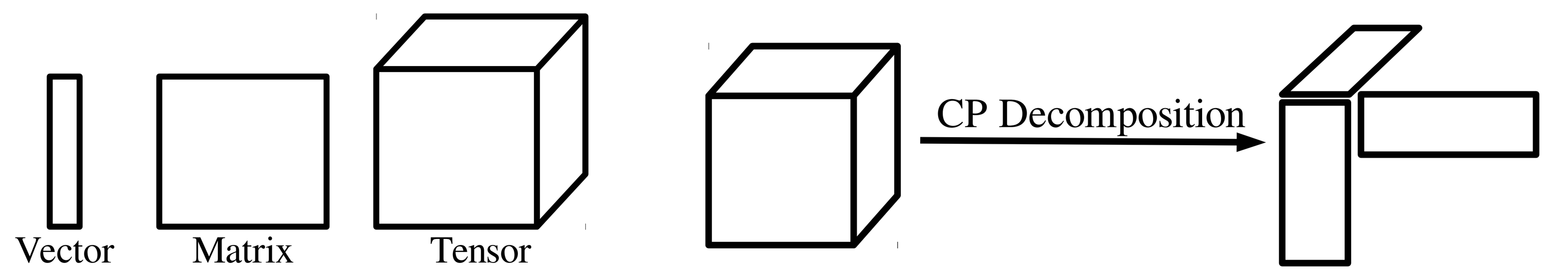
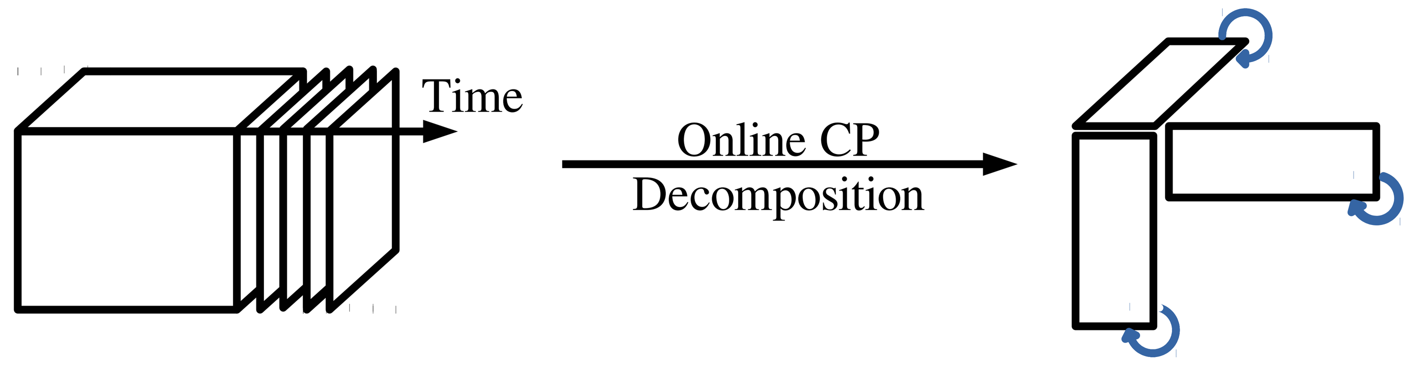
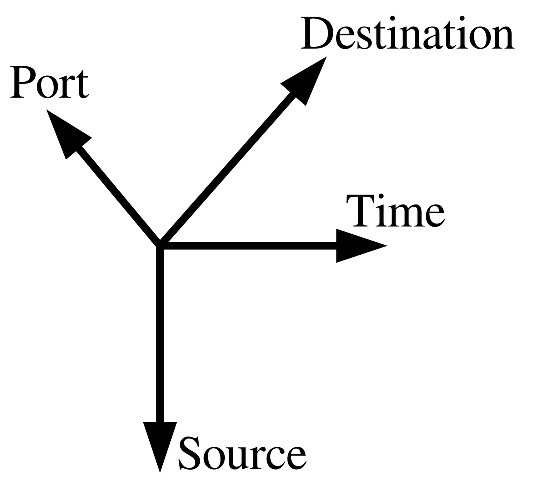
Introduction



- **Tensors** (multi-way array) are a natural representation for multi-dimensional data, e.g. videos, time-evolving networks
- Similar to PCA and SVD, **CP decomposition** is a method to simplify and summarize tensors
- **Online CP decomposition** is an extension of CP decomposition to **online tensors** (tensors that grow along the time dimension)



A Motivating Example



- Network traffic data
- Multi-way structure ⇒ tensor representation
- Highly complex data ⇒ simplified by CP decomposition
- New data keeps arriving ⇒ online CP is required

Notations

Symbol	Meaning
$\mathbf{a}, \mathbf{A}, \mathcal{X}$	vector, matrix, tensor
$\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$	N^{th}-order tensor
$\mathbf{A}^\top, \mathbf{A}^{-1}, \mathbf{A}^\dagger, \lVert\mathbf{A}\rVert$	transpose, inverse, pseudoinverse and Frobenius norm of $\mathbf{A}$
$\mathbf{A}^{(1)}, \mathbf{A}^{(2)}, \ldots, \mathbf{A}^{(N)}$	a sequence of N matrices
$\odot, \circledast, \oslash$	Khatri-Rao product, element-wise multiplication and division
$\mathbf{X}_{(n)}$	mode-n unfolding of tensor $\mathcal{X}$
$[\![\bullet]\!]$	CP decomposition operator

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Method

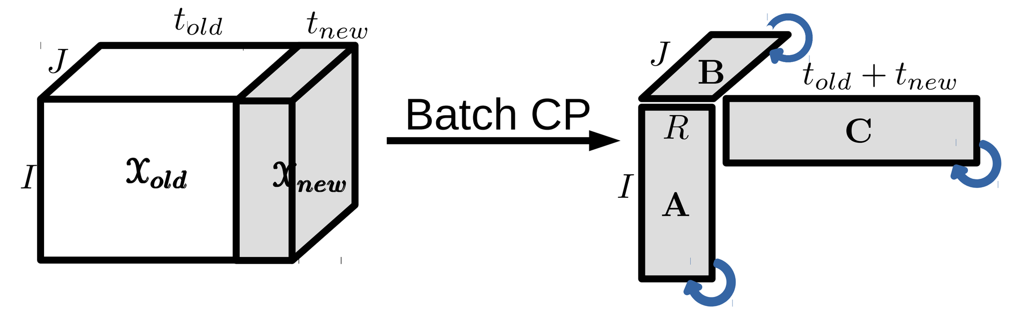
*We illustrate our method with a 3^{rd}-order example, higher-order case is addressed in a similar way and more detail can be found in our paper.

Problem

Given: $\mathcal{X}_{old} \in \mathbb{R}^{I \times J \times t_{old}}$, its CP decomposition $[\![\mathbf{A}_{old}, \mathbf{B}_{old}, \mathbf{C}_{old}]\!]$, and $\mathcal{X}_{new} \in \mathbb{R}^{I \times J \times t_{new}}$
Find: CP decomposition $[\![\mathbf{A}, \mathbf{B}, \mathbf{C}]\!]$ of $\mathcal{X} \in \mathbb{R}^{I \times J \times (t_{old}+t_{new})}$, where $t_{old} \gg t_{new}$

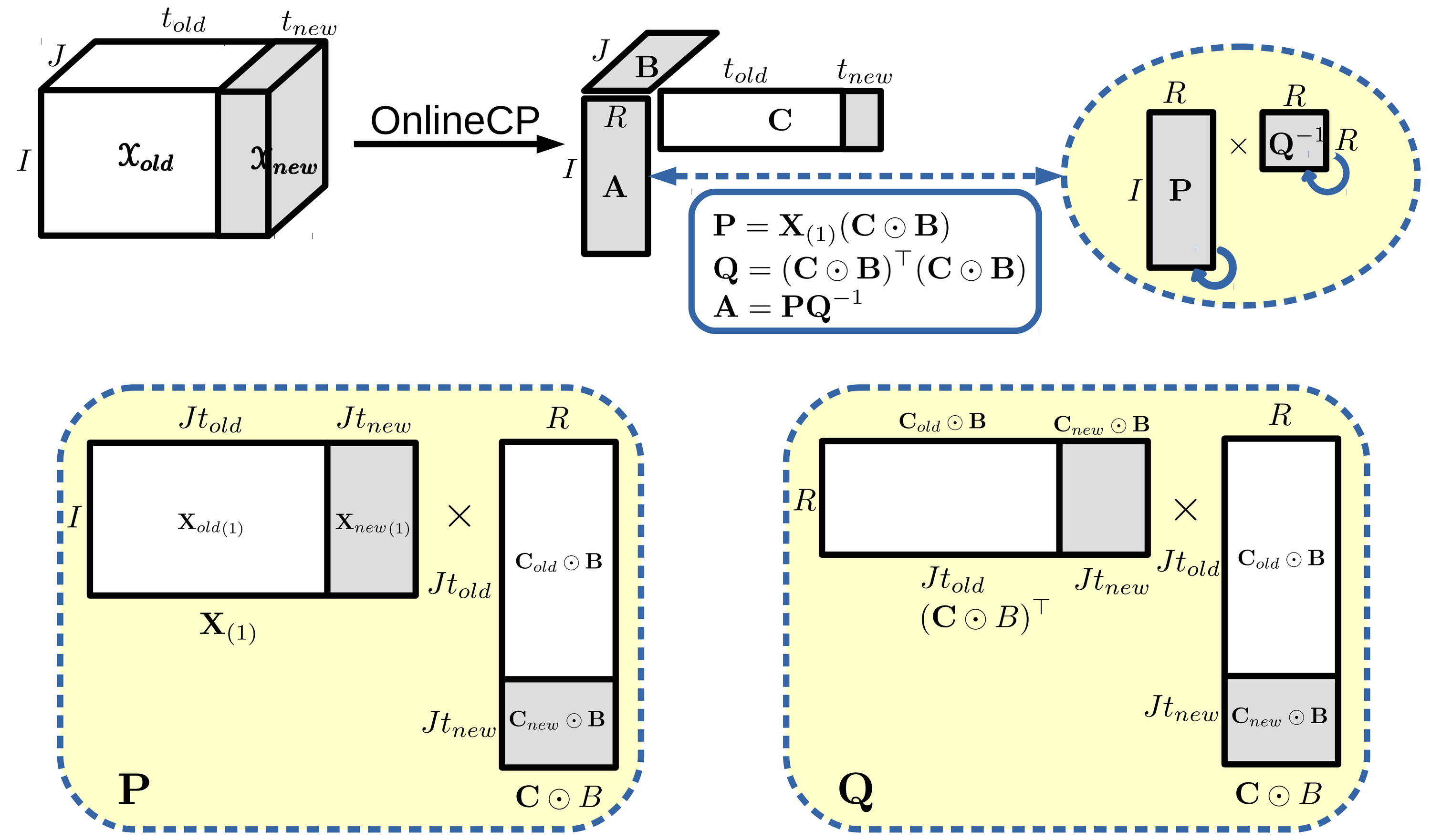
Batch Method

- Directly applying *Alternating Least Square* (ALS) to decompose $\mathcal{X}$
- Online tensor ⇒ using **previous decomposition as initialization**
- However, still **computationally expensive**

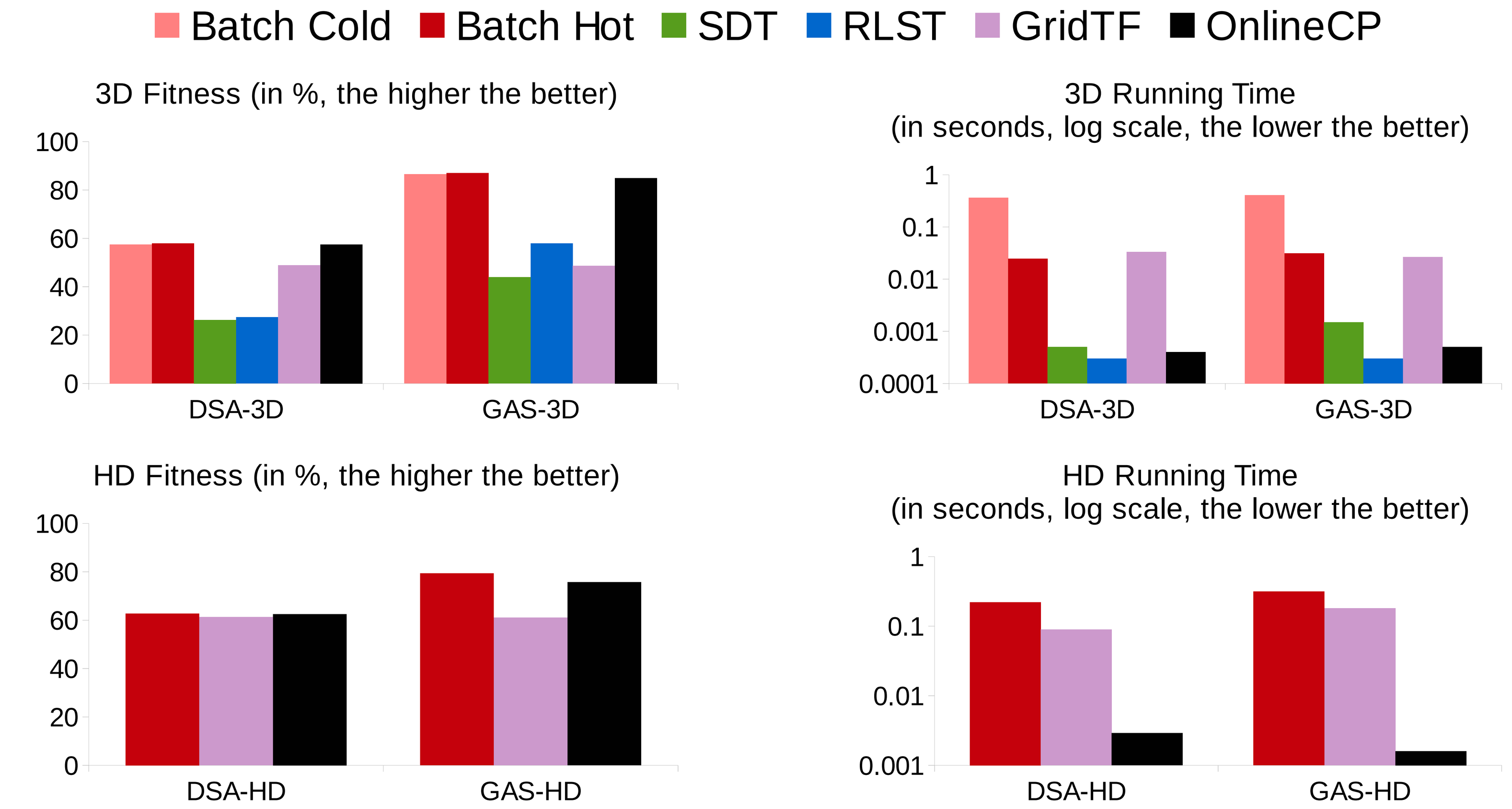


OnlineCP

- ALS like algorithm
- For **time** mode, projecting $\mathbf{X}_{new(3)}$ to $\mathbf{C}$ by fixing $\mathbf{A}$ and $\mathbf{B}$
- For **non-temporal** modes $\mathbf{A}$ and $\mathbf{B}$, using **complementary matrices** to temporally store the useful information of the previous time step; then **incrementally** updating complementary matrices, and further estimating loading matrices

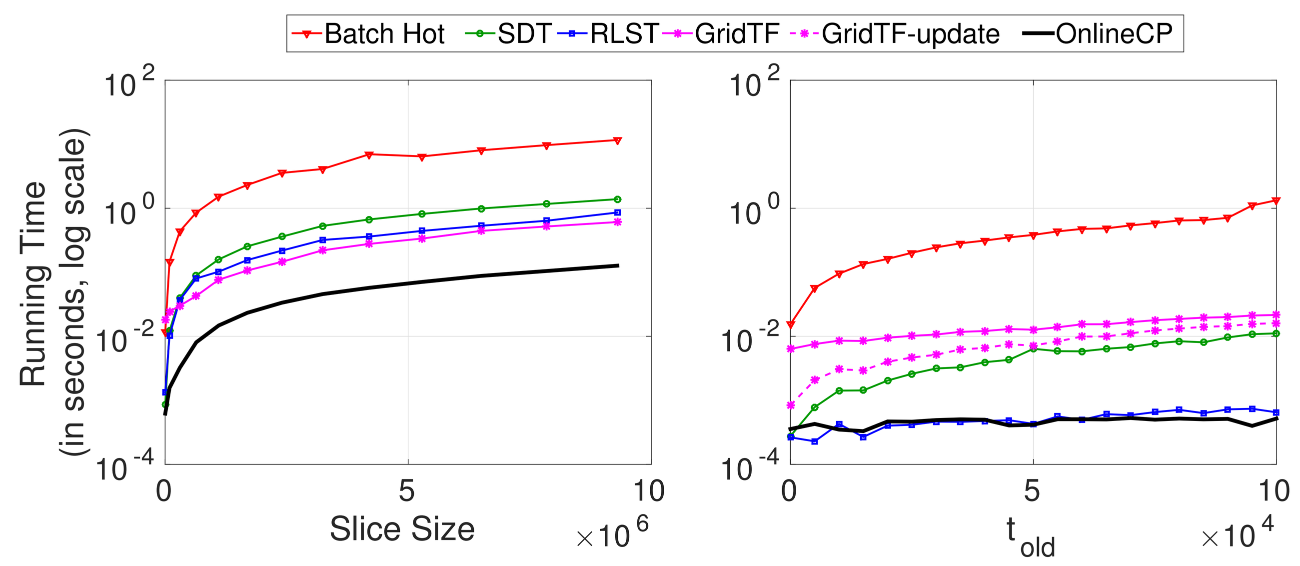


Results — Effectiveness & Efficiency



- Effectiveness:
  1. **Batch** methods always obtain the **best** fitness overall
  2. **OnlineCP** shows **comparable** results to batch algorithms
  3. Performance of **other online algorithms** is **much worse**
- Efficiency:
  1. **Batch** methods are **extremely time-consuming**
  2. Improvement of **GridTF** is **not significant**
  3. Both **SDT** and **RLST** show **very good** performance
  4. **OnlineCP** is also **quite efficient** and **significantly faster** than batch methods

Results — Scalability



- All approaches linearly increase w.r.t. **slice size**, while the growth of **OnlineCP** is much slower than others
- **Batch**, **SDT** and **GridTF** show linear increase w.r.t. t_{old}, whereas the running time of **OnlineCP** and **RLST** are constant

Conclusion

Compared with existing work for online CP decomposition, our proposed **OnlineCP** algorithms,

1. is applicable to both 3^{rd}-order and higher-order tensors
2. is able to gain comparable result to batch methods
3. shows better performance than existing online approaches in terms of effectiveness, efficiency and scalability